Structural Mechanics (1)

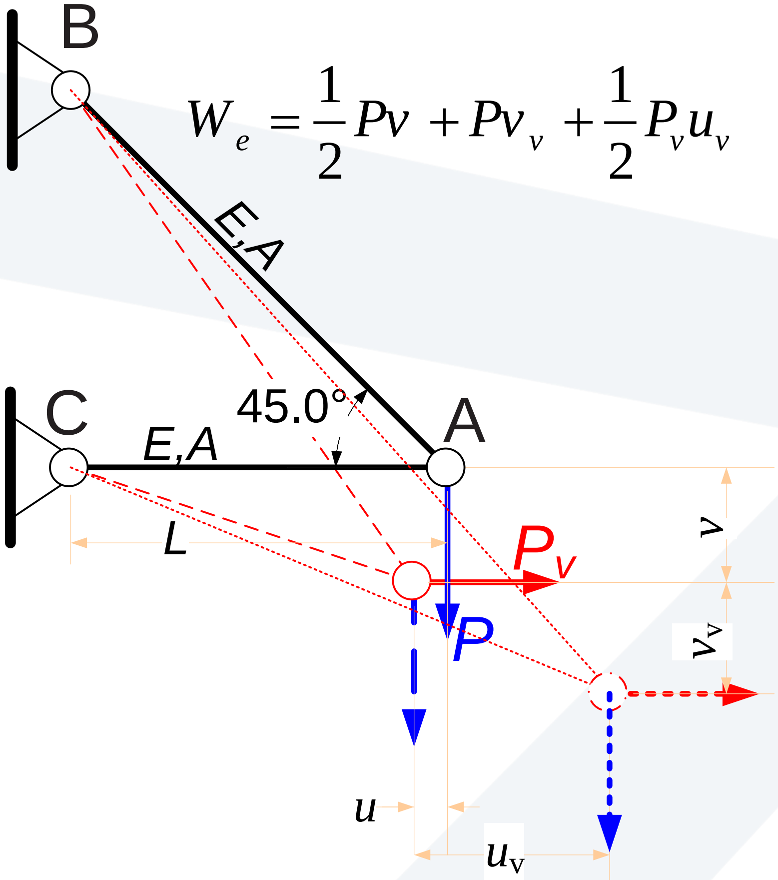
Week No-03

Deflection in Determinate Structures

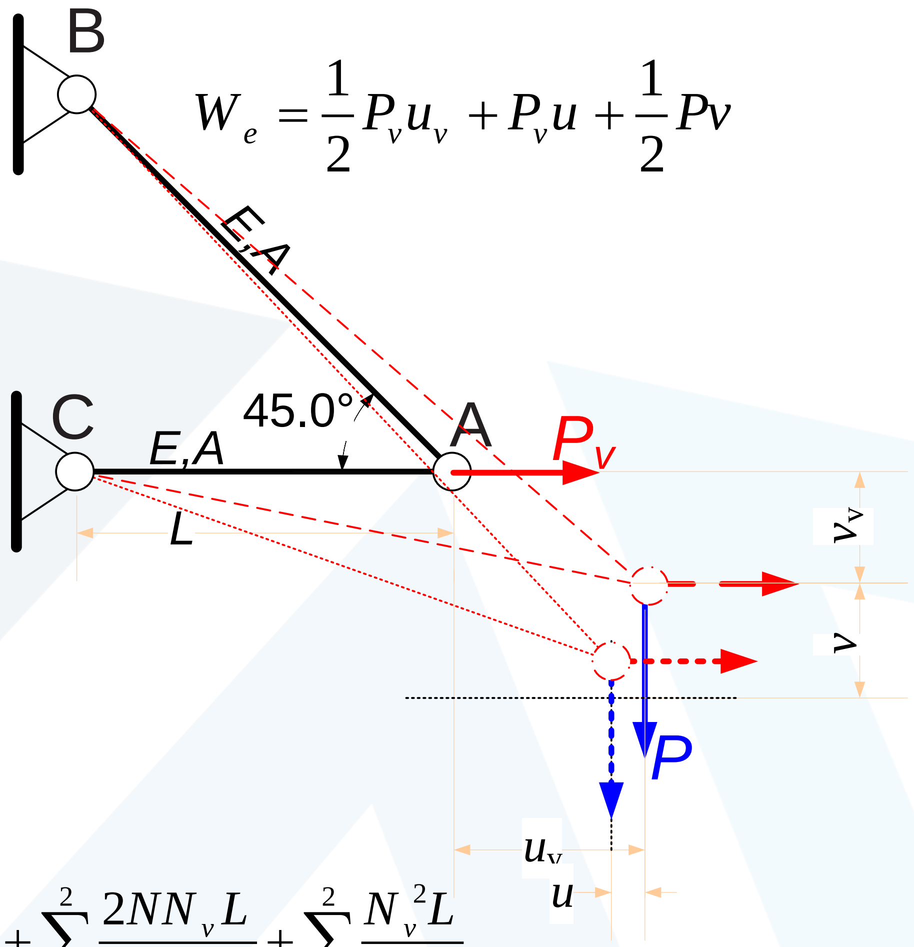
Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Deflections of Trusses by the Virtual Work Method



$$W_e = \frac{1}{2}Pv + Pv_v + \frac{1}{2}P_v u_v$$



$$W_e = \frac{1}{2}P_v u_v + P_v u + \frac{1}{2}Pv$$

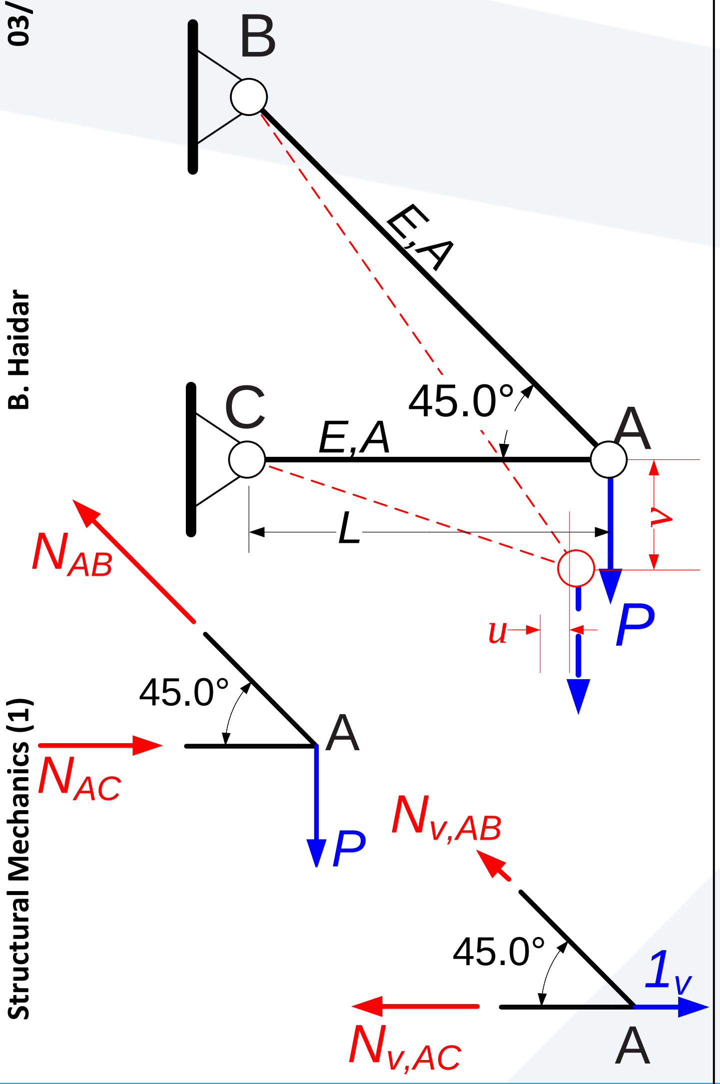
$$U = \sum_{i=1}^{2} \frac{(N + N_v)^2 L}{2EA} = \sum_{i=1}^{2} \frac{N^2 L}{2EA} + \sum_{i=1}^{2} \frac{2NN_v L}{2EA} + \sum_{i=1}^{2} \frac{N_v^2 L}{2EA}$$

$$W_e = U \Rightarrow Pv_v = P_v u = \sum_{i=1}^{2} \frac{NN_v L}{EA}$$

Making $P_v=1$

$$(1_v)u = u = \sum_{i=1}^{2} \frac{NN_v L}{EA}$$

The truss shown in Fig. carries a gradually applied load P at the joint A. **Compute the horizontal deflection u at A.**



1) Analyzing the truss under the real load, for N in the two members. We found

$$N_{AB}=1.41P\ (T) \qquad N_{AC}=P\ (C)$$

2) Applying a virtual unit load at A in the direction of u.

Then Analyzing For N_v

Considering the vertical equil. at A:

$$N_{v,AB}\cos 45^{o}=0 \Rightarrow N_{v,AB}=0$$

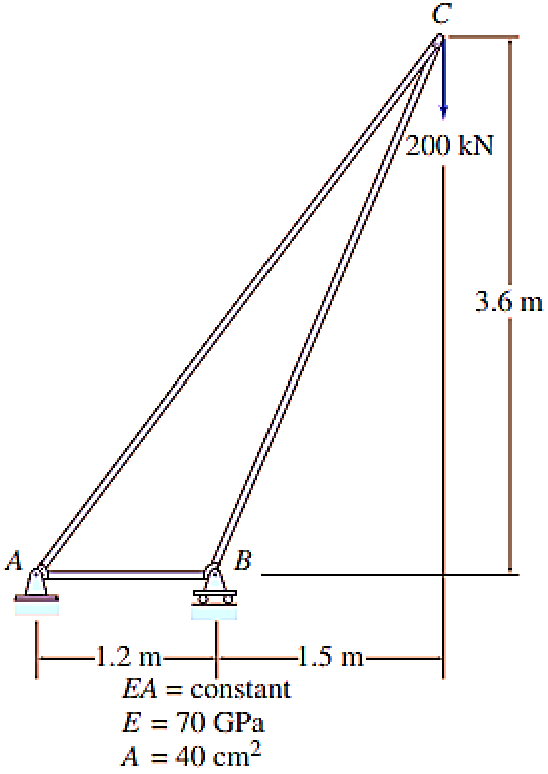
Considering the horizontal equil. at A:

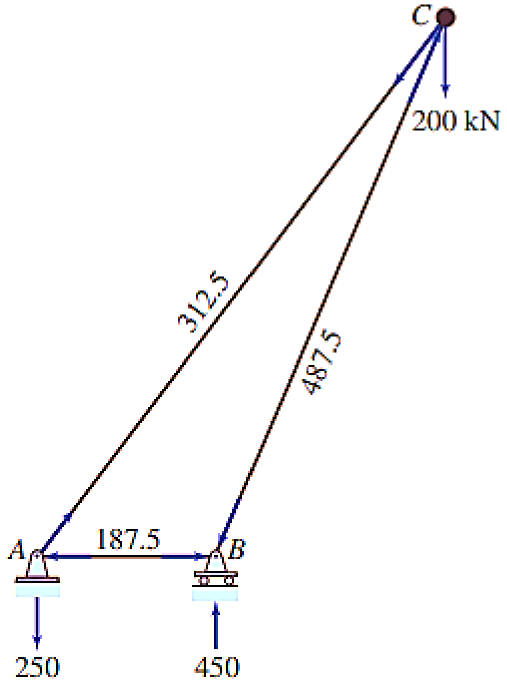
$$-N_{v,AC}+1_v=0 \Rightarrow N_{v,AC}=1\ (T)$$

3) Applying the V.W.M

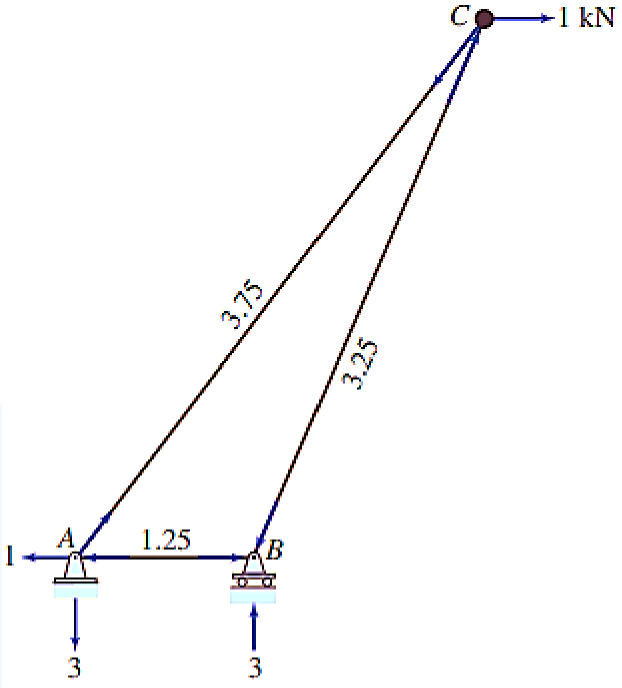
$$(1_v)u=u=\sum_{i=1}^{2}\frac{NN_vL}{EA}=0+\frac{(-P)(+1)L}{EA}=\frac{-PL}{EA}$$

Example 01: Use the virtual work method to determine the horizontal components of the deflection at joint C of the truss shown in the following figure.





Real System — F Forces



) Virtual System — F_v Forces

Example 01: The member axial forces due to the real system (N) and this virtual system (N_{v1}) are then tabulated as shown in the following table:.

Member	L (m)	N (kN)	N_v (kN)	$N_v(NL)$ ($kn^2.m$)
AB	1.2	-187.5	-1.25	281.25
AC	4.5	312.5	3.75	5273.44
BC	3.9	-487.5	-3.25	6179.06
				11733.75

Example 01: the virtual work expression is applied to determine Δ_{CH} as shown below:

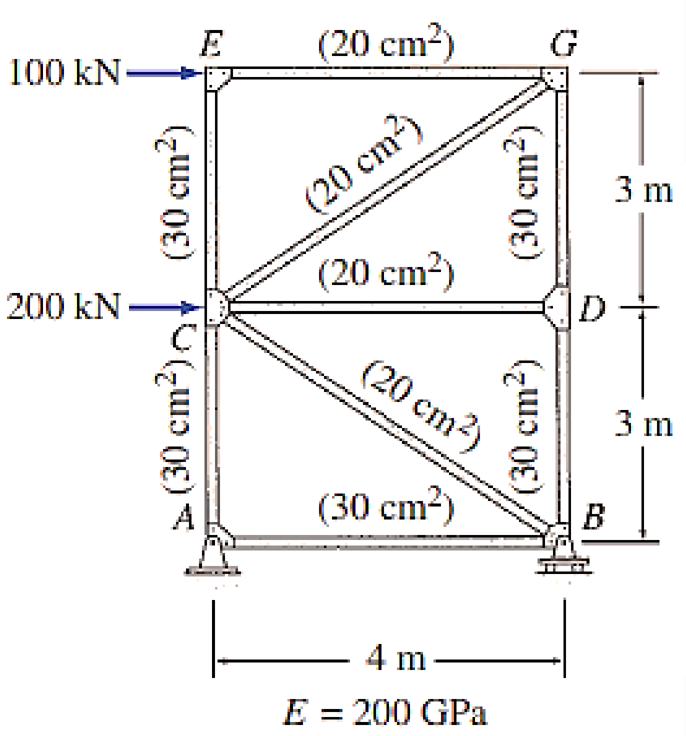
$$1(\Delta_{CH}) = \frac{1}{E}\sum\frac{N_v(NL)}{A}$$

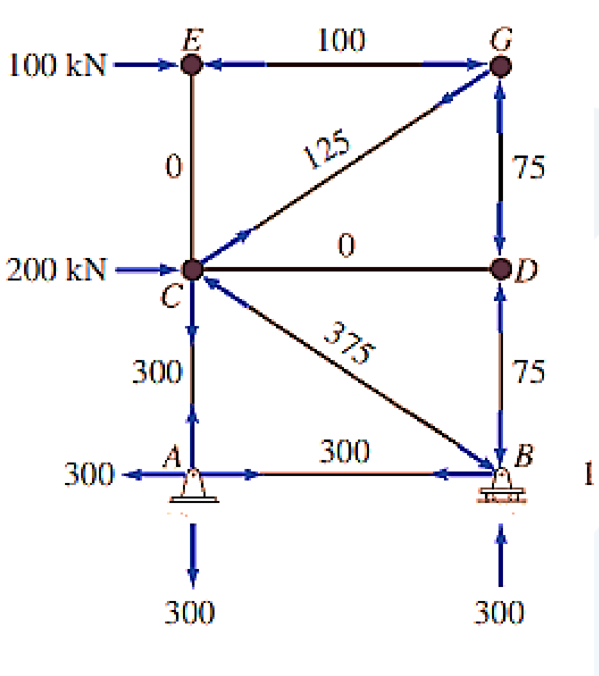
$$(1\text{kN})(\Delta_{CH}) = \frac{11{,}733.75}{70(10^6)4000(10^{-6})}\text{kN.m}$$

$$\Delta_{CH} = 0.042\text{ m}$$

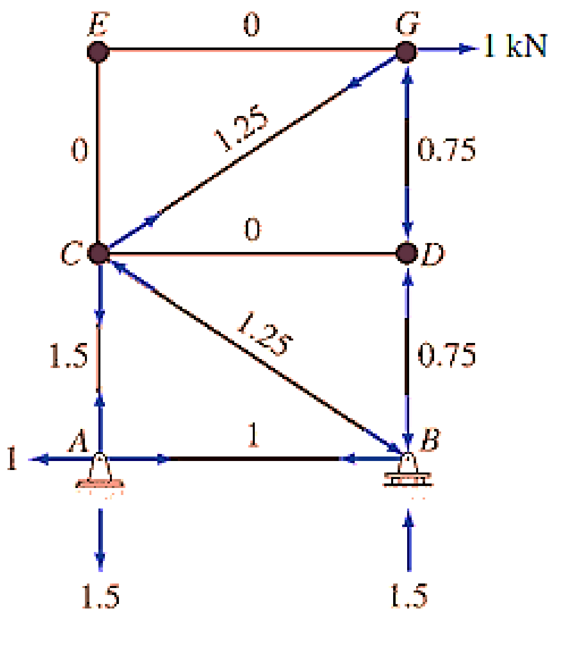
$$\Delta_{CH} = 42\ mm\ \rightarrow$$

Example 02: Use the virtual work method to determine the horizontal components of the deflection at joint G of the truss shown in the following figure.





(b) Real System — F Forces



(c) Virtual System — F_v Forces

Example 02: The member axial forces due to the real system (N) and this virtual system (N_{v1}) are then tabulated as shown in the following table:

Member	L (m)	A (m^2)	N (kN)	N_v (kN)	N_{v1}(NL/A) (kn^2/m)
AB	4	0.003	300	1	400000
CD	4	0.002	0	0	0
EG	4	0.002	-100	0	0
AC	3	0.003	300	1.5	450000
CE	3	0.003	0	0	0
BD	3	0.003	-75	-0.75	56250
DG	3	0.003	-75	-0.75	56250
BC	5	0.002	-375	-1.25	1171875
CG	5	0.002	125	1.25	390625
					2525000

Example 02: the virtual work expression is applied to determine Δ_{GH} as shown below:

$$1(\Delta_{GH}) = \frac{1}{E}\sum \frac{N_v(NL)}{A}$$

$$(1\ kN)(\Delta_{CH}) = \frac{2525000}{200(10^6)}kN.m$$

$$\Delta_{GH} = 0.0126 \text{ m}$$

$$\Delta_{GH} = 12.6\ mm\ \rightarrow$$

Example 03: Use the virtual work method to determine the horizontal and vertical components of the deflection at joint B of the truss shown in the following figure. **Then find the vertical deflection at D without V.U.L.**